Optimal Control for Articulated Soft Robots

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Introduction (1)

- Soft robots can execute tasks with safer interactions. However, controllers that can exploit the systems' capabilities are still missing.
- Differential dynamic programming (DDP) has emerged as a promising tool for achieving highly dynamic tasks [1].
- We propose an efficient DDP-based algorithm for trajectory optimization of articulated soft robots that can optimize the state trajectory, input torques, and stiffness profile [2].

Model of Soft Articulated Arm (2)

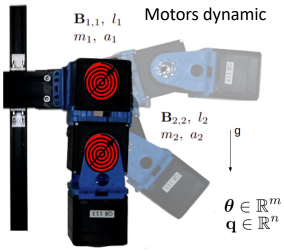
Links dynamic

$$\mathbf{M}(\mathbf{q}(t))\ddot{\mathbf{q}}(t) + \mathbf{C}(\mathbf{q}(t),\dot{\mathbf{q}}(t))\dot{\mathbf{q}}(t) + \mathbf{G}(\mathbf{q}(t)) + \mathbf{K}(t)(\mathbf{q}(t) - \mathbf{S}\boldsymbol{\theta}(t)) = \mathbf{0},$$

Motors dynamic

$$\mathbf{B}\ddot{\boldsymbol{\theta}}(t) + \mathbf{S}^\top\mathbf{K}(t)(\mathbf{S}\boldsymbol{\theta}(t) - \mathbf{q}(t)) - \boldsymbol{\tau}(t) = \mathbf{0},$$

Inertia Matrix	$\mathbf{M}(\mathbf{q}) \in \mathbb{R}^{n\times n}$
Coriolis Matrix	$\mathbf{C}(\mathbf{q}) \in \mathbb{R}^{n\times n}$
Gravity Vector	$\mathbf{G}(\mathbf{q}) \in \mathbb{R}^{n}$
Stiffness Matrix	$\mathbf{K} \in \mathbb{R}^{n\times n}$
Damping Matrix	$\mathbf{D} \in \mathbb{R}^{n\times n}$
Control Inputs	$\boldsymbol{\tau} \in \mathbb{R}^{m}$
Actuation Matrix	$\mathbf{S} \in \mathbb{R}^{n\times m}$
Inertia Matrix	$\mathbf{B} \in \mathbb{R}^{m\times m}$



Optimal Control (3)

Optimal control problem:

$$\min_{(\mathbf{q}_s,\dot{\mathbf{q}}_s,\boldsymbol{\theta}_s,\dot{\boldsymbol{\theta}}_s),(\boldsymbol{\tau}_s)} \ell(\mathbf{q}_N,\dot{\mathbf{q}}_N,\boldsymbol{\theta}_N,\dot{\boldsymbol{\theta}}_N) + \sum_{k=0}^{N-1}\int_{t_k}^{t_{k+1}} \ell(\mathbf{q}_k,\dot{\mathbf{q}}_k,\boldsymbol{\theta}_k,\dot{\boldsymbol{\theta}}_k,\boldsymbol{\tau}_k)dt,$$

$$\text{s.t.}\quad [\mathbf{q}_{k+1},\dot{\mathbf{q}}_{k+1},\boldsymbol{\theta}_{k+1},\dot{\boldsymbol{\theta}}_{k+1}] = \psi(\mathbf{q}_k,\dot{\mathbf{q}}_k,\ddot{\mathbf{q}},\boldsymbol{\theta}_k,\dot{\boldsymbol{\theta}}_k),$$
$$[\ddot{\mathbf{q}}_k,\ddot{\boldsymbol{\theta}}_k] = \mathrm{FD}(\mathbf{q}_k,\dot{\mathbf{q}}_k,\boldsymbol{\theta}_k,\dot{\boldsymbol{\theta}}_k,\boldsymbol{\tau}_k),$$
$$[\mathbf{q}_k,\boldsymbol{\theta}_k] \in \mathcal{Q},\ [\dot{\mathbf{q}}_k,\dot{\boldsymbol{\theta}}_k] \in \mathcal{V},\ \boldsymbol{\tau}_k \in \mathcal{U},$$

$\psi(\cdot)$ Numerical Integrator
$\mathrm{FD}(\cdot)$ Forward Dynamics
$\mathcal{Q},\ \mathcal{V}$ Constraint Set

DDP (4)

Solves the Bellman optimal backward (time).

$$V(\mathbf{x}_k) = \min_{\mathbf{u}_k}\{l_k(\mathbf{x}_k,\mathbf{u}_k) + V_{k+1}(\mathbf{f}(\mathbf{x}_k,\mathbf{u}_k))\}$$

II-order approximation

$$\Delta V \approx \frac{1}{2}\begin{bmatrix}1\\ \delta\mathbf{x}_k\\ \delta\mathbf{u}_k\end{bmatrix}^\top \begin{bmatrix}0 & Q_{\mathbf{x}_k}^\top & Q_{\mathbf{u}_k}^\top\\ Q_{\mathbf{x}_k} & Q_{\mathbf{xx}_k} & Q_{\mathbf{xu}_k}\\ Q_{\mathbf{u}_k} & Q_{\mathbf{ux}_k} & Q_{\mathbf{uu}_k}\end{bmatrix}\begin{bmatrix}1\\ \delta\mathbf{x}_k\\ \delta\mathbf{u}_k\end{bmatrix}$$

Backward Pass

$$\delta\mathbf{u} = \arg\min_{\delta\mathbf{u}} \mathbf{Q}(\delta\mathbf{x},\delta\mathbf{u}) = -\hat{\mathbf{k}} - \hat{\mathbf{K}}\delta\mathbf{x}$$
$$\hat{\mathbf{k}} = \mathbf{Q}_{\mathbf{uu}}^{-1}\mathbf{Q}_{\mathbf{u}} \qquad \hat{\mathbf{K}} = \mathbf{Q}_{\mathbf{uu}}^{-1}\mathbf{Q}_{\mathbf{ux}}$$

Forward Pass

$$\hat{\mathbf{u}}_\mathbf{k} = \mathbf{u}_\mathbf{k} + \alpha\hat{\mathbf{k}} + \hat{\mathbf{K}}(\hat{\mathbf{x}}_\mathbf{k} - \mathbf{x}_\mathbf{k})$$
$$\hat{\mathbf{x}}_{\mathbf{k}+1} = \mathbf{f}_\mathbf{k}(\hat{\mathbf{x}}_\mathbf{k},\hat{\mathbf{u}}_\mathbf{k})$$

Analytical Derivatives (5)

Forward Dynamics

$$\begin{bmatrix}\ddot{q}\\ \ddot{\theta}\end{bmatrix} = \begin{bmatrix}M(q) & 0\\ 0 & B_\mathrm{m}\end{bmatrix}^{-1}\begin{bmatrix}\tau_\mathrm{l}\\ \tau_\mathrm{m}\end{bmatrix}$$

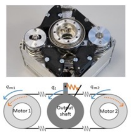
$$\tau_\mathrm{l} = -C(q,\dot{q})\dot{q} - D\dot{q} - \frac{\partial U(q,\theta)}{\partial q} + G(q)$$
$$\tau_\mathrm{m} = -D_\mathrm{m}\dot{\theta} - \frac{\partial U(q,\theta)}{\partial \theta} + S\tau$$

Analytical Derivatives

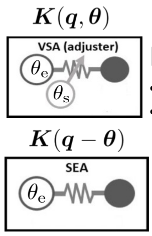
$$\begin{bmatrix}\delta\ddot{q}\\ \delta\ddot{\theta}\end{bmatrix} = -\begin{bmatrix}M(q) & 0\\ 0 & B_\mathrm{m}\end{bmatrix}^{-1}\left(\begin{bmatrix}\frac{\partial\tau_\mathrm{l}}{\partial x}\\ \frac{\partial\tau_\mathrm{m}}{\partial x}\end{bmatrix}\delta x + \begin{bmatrix}\frac{\partial\tau_\mathrm{l}}{\partial u}\\ \frac{\partial\tau_\mathrm{m}}{\partial u}\end{bmatrix}\delta u\right)$$

Experimental Setup (6)

Qb Move [3]

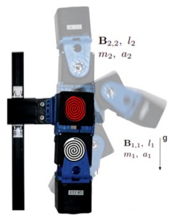


Agonist-Antagonist



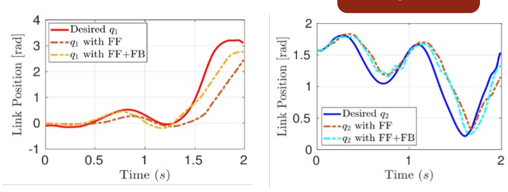
Experiments:
- 4 DoFs SEAS/VSAs
- 2 DoFs VSAs/SEAs

Active SEA
Passive SEA

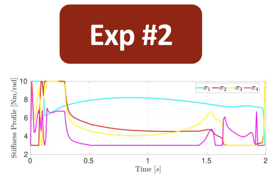


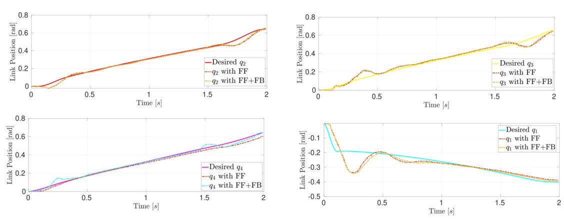
Results (6)

Exp #1



Exp #2





Conclusion

We proposed an efficient way to compute the dynamics and analytical derivatives of soft articulated. The state-feedback controller based on local and optimal policies from Box-FDDP/FDDP helped to improve all tasks

Future Works

Future work will include **MPC solutions**.

References

[1] Mastalli, C., et al. (2020, May). Crocoddyl: An efficient and versatile framework for multi-contact optimal control. In 2020 IEEE International Conference on Robotics and Automation (ICRA) (pp. 2536-2542). IEEE.
[2] Vanderborght, B., et al. (2006). Exploiting natural dynamics to reduce energy consumption by controlling the compliance of soft actuators. IJRR, 25(4), 343-358.
[3] Catalano, M. G., et at. (2011, May). VSA-CubeBot: A modular variable stiffness platform for multiple degrees of freedom robots. In 2011 IEEE ICRA (pp. 5090-5095). IEEE.